

## Depth control of Submarine: a Fuzzy Logic approach

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**Abstract:** Ocean has attracted the attention of the researchers owing to its importance on environmental issues, resources, scientific works and military tasks requiring critical control operations of the submarine to achieve the desired objectives. This paper represents the performance comparison of various controllers for the depth control of the submarine employing the stern plane actuation technique which actuates the stern plane motor to achieve the desired depth. The controllers such as PI Controller, Fuzzy logic based Controller (FLC) and Type 2 Fuzzy Logic based Controllers (T2FLC) have been applied for the depth control operation of the submarine and the simulation has been carried out based on quantitative performance analysis of the controllers using various performance criteria in MATLAB Simulink<sup>®</sup> environment. The results obtained in terms of Stern plane angle, Rate of change of Depth and Actual depth, indicate that the T2FLC provides better performance than other controllers used.

**Keywords:** Submarine; Depth control; Stern Plane; Fuzzy Logic Control; Type-2 Fuzzy Logic Control.

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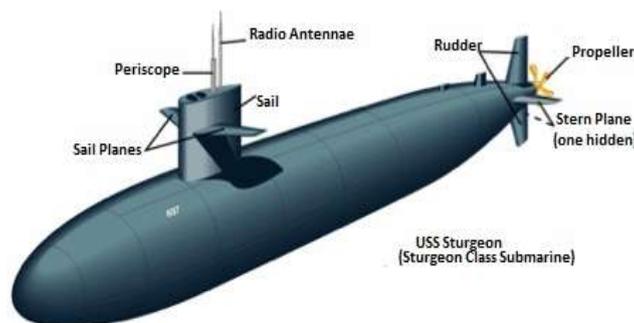
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### I. Introduction

A Submarine is a watercraft capable of independent operation underwater deriving its origin from “Bathyscaphe”, which is evolved from the “Diving Bell” and is commonly, refers to Remotely Operated Vehicles (ROVs) and Robots, as well as medium sized or smaller vessels, such as the Midget Submarine and the Wet Sub. Submarines were first widely used during World War-1(1914-1918), and now figure in many Navies large and small. Civilian uses for Submarine include marine science, salvage, exploration and facility inspection and maintenance. Submarines can also be modified to perform more specialized functions such as search and rescue missions or underwater cable repair. Military usage includes attacking enemy surface ships, submarines, aircraft carrier protection, blockade running and ballistic missile submarines as a part of nuclear strike force, reconnaissance, conventional land attack and covert insertion of Special Forces. Submarines are also used in tourism and for undersea archeology. Modern submarines are normally designed mostly with submerged operation in mind and these have an inner pressure hull and an outer streamlined hull. For security reasons submarines normally surface only on leaving or returning to base [6]. Depth control of the submarine, in general, signifies the operation of the submarine in which it dives to a certain depth and stays there with all the control surfaces working properly. Submarines were, and are, very vulnerable to accidents which prevent the boat from surfacing [1]. It is an important aspect in the operational effectiveness of the submarine as the submarine in its operational life mainly operates underwater with its depth changing as per the need of the crew handling it. In order to make Submarines complete tasks such as navigation at set depth, dive or rise quickly and quietly underwater the automatic control of such vehicles presents several difficulties due to non linearity in dynamics, the presence of unpredictable external disturbances and the high uncertainty level in the model. The operation becomes even more complicated in the event of system failures, such as a stern plane jam. When the submarine is operated at periscopic depth in littoral water, extremely precise depth control and pitch control is required because the water is very shallow and the wave effect is very significant [8]. The hydroplanes are used for controlling the lift or driving forces [6]. Systems which are acting under the sea must reach reference depth with minimum



**Figure 1.1** USS Sturgeon (Sturgeon Class Submarine)

oscillations in a short time. Oscillations the reference depth exerts extra pressure on submarine. So, the system depth must be checked continuously with a controller [7]. Depth control of underwater vehicles is usually done by using control surfaces, thrusters and ballast systems. For a neutrally buoyant vehicle, stern rudders are attractive for diving and depth changing maneuvers, since they require relatively little control energy compared to thrusters.

In literature there are many studies about the depth control of the Submarine and controller designs. During 2<sup>nd</sup> world war, the formidable U-boat force of Germany laid waste to the Allied Naval Force [1] and earlier depth-control autopilots were found in German submarines at the end of the 2<sup>nd</sup> World War [2]. It is known that French submarines included improved versions of the German designs after the war [3]. These earlier versions included controllers which are typically single-input single-output (SISO) Proportional-Derivative (PD) type designs in which bow and stern hydroplanes are geared together. The control surfaces for a submarine include a rudder, a set each of stern, sail and bow hydroplanes [4]. The rudder on the vessel is of course for controlling the yaw motions. The hydroplanes are used for controlling the lift or diving forces and also for controlling the trim angle. The stern planes on a vessel are usually much larger than those of bow planes [6]. Submarine is required to keep the necessary navigating pose (depth, roll angle and trim angle) for the reason of concealment [11]. A LabVIEW based submarine depth control simulator with PID and FLC was proposed by M. Ekici *et al.* In which; in order to keep the submarine at a certain level below the sea surface, a FLC as a major control unit was designed and employed with LabVIEW Control Design and Simulation Toolkit. Also comparison between FLC and PID controlled system was presented and results showed that system had better settling time and no overshoots with FLC [7]. Roger Xu *et al.* proposed submarine pitch and depth control using FCMAC (Fuzzy Cerebellar Model Airthematic Computer) neural networks in which they developed a non-model based nonlinear adaptive control scheme to control the depth and the pitch of submarines operating in shallow water [8]. This fact motivated the research on multivariable depth-keeping control systems; it is known that the 'split plane mode', i.e., the independent use of bow and stern hydroplanes improves the performance of depth-keeping controllers. E. Liceaga-Castro *et al.* proposed  $H_\infty$  controllers to solve the depth keeping problem, combining polynomial, and state space  $H_\infty$  methods [9]. A Submarine multivariable Depth control System was developed by E. Liceaga-Castro and G.M. Van der Molen using classical methods allowing the control system design with its robustness characteristics expressed in terms of actual gain and phase margins [10]. Lionel Lapiere addressed the Robust diving control of the Submarine where a diving control design based on Lyapunov theory and back stepping techniques was verified. By using adaptive and switching schemes, the control system was able to meet the required robustness in diving control [12].  $H_\infty$  submarine depth and pitch control was proposed by G.M. Van der Molen *et al.* a multivariable ( $2 \times 2$ )  $H_\infty$  controller was designed for depth keeping of a submarine under wave disturbances using bow and stern hydroplanes. The final controller was functioning well while the submarine operated at periscope depth under a heavy sea, over a wide speed range [13]. S.K. Lee *et al.* presented a 6-DOF Manta type unmanned underwater test vehicle with its diving and steering controls being governed by mathematical model PID and Sliding mode control [14]. Another efficient multivariable system was designed by E. Liceaga-Castro *et al.* where analysis and design of multivariable depth keeping controllers for a class of submarines was studied [15]. Biomimetic locomotion was suggested by K.H. Low for a submersible vehicle and its depth control was studied [16]. Another comparative study was put forth by R. Allen *et al.* in which comparison between classical controller, FLC and sliding mode control was done and concluded that no single technique appears promising but every technique has its own advantages (e.g. performance) and disadvantages (e.g. complexity) that designers need to consider carefully with skill and judgment to produce desirable results [17].

This paper is organized as follows: Section 2 explains the modeling of the submarine dynamics for Depth control by Stern plane actuation. Various controllers used for Depth control are discussed in Section 3. Results and conclusions are discussed in sections 4 and 5 respectively followed by References.

## II. Submarine Dynamics for Depth Control

The type of boat considered here has stern hydroplane as the control surface. The submarine model is describing a realistic submarine with the Stern plane mounted horizontally. Figure 2.1 shows depth control of a submarine using stern plane angle which governs the diving and depth keeping operation of the submarine. The submarine dynamics for the Depth Control operation is represented by a simple first order model as,

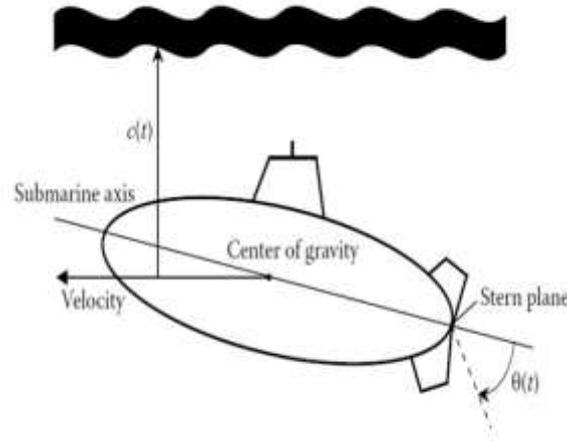


Figure 2.1 Depth control of submarine using Stern plane actuation

$$\tau \frac{dv}{dt} + v = K_{\theta} \frac{d\theta}{dt} + K_{\theta} \theta \tag{2.1}$$

Where,  $v = v(t)$ ; is the depth rate of the Submarine.

Integrating the depth rate yields the depth of the submarine

$$c = \int v dt \tag{2.2}$$

The error signal is output from the summer as

$$e = r - c \tag{2.3}$$

The baseline parameter values for submarine dynamics given in equation (1) are taken as

$\tau = 10$  s;  $K_{\dot{\theta}} = 20$  ft/s per deg/s;  $K_{\theta} = 10$  ft/s per deg

A simplified block diagram of the depth control system is shown in figure 2.2.

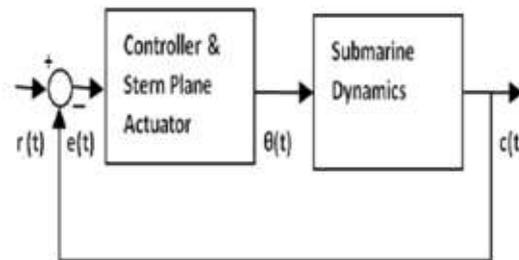


Figure 2.2 Block diagram of a submarine depth control system

Depth control usually involves the control of Stern Plane angle (the angle between the longitudinal axis of the submarine and the horizontal plane). To control this two set of actuators or control surfaces are available: the bow hydroplanes and the stern hydroplanes [4]. The error signal  $e(t)$  is the difference between commanded depth  $r(t)$  and the actual depth  $c(t)$ . It is fed back to the controller that sends a signal to the stern plane actuator motor angle  $\theta(t)$ . The submarine depth responds to changes in the stern plane angle [5].

## III. Design of Controllers

To control a process variable some control strategies are essential. A control strategy consists of two aspects:

- a) Control configuration; and
- b) Controller.

Control configuration can be further categorized as: feedback control configuration, feed forward control configuration, cascade control configuration etc. The second aspect of a control strategy is controller, which

gives actuating signal to an actuator on the basis of computed error, which is calculated from the set point and measured value of process variable.

In this paper, a simple feedback configuration is utilized while using various controllers such as conventional PI, FLC (Mamdani Type) and T2FLC (Mamdani Type) are discussed in following subsections.

### 3.1 Proportional-Integral (PI) Controller

In industry, most of control systems are conventional P/PI/PID controllers, because they provide a simple, cost-effective and robust control for most systems. In the present work conventional PI controller is considered because derivative action is obsolete in the case of this system, and it is also proved by the fact that controller optimization gives a very small derivative action which can be neglected. A basic block diagram of a PI Controller is given in Figure 3.1 the mathematical expression of the conventional PI in position form is given as:

$$u_{PI}(t) = K_p e(t) + K_I \int_{-\infty}^t e(\tau) d\tau \quad (3.1)$$

Where  $u(t)$  is controller output,  $e(t)$  is error,  $K_p$  is proportional gain, and  $K_I$  is Integral gain.

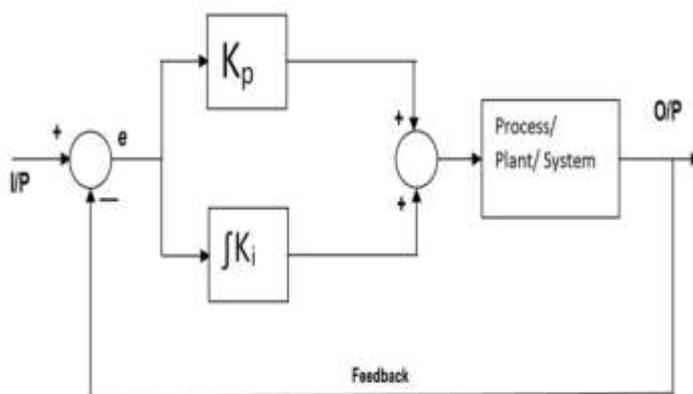


Figure 3.1 Block diagram of PI Controller

But in present study, conventional PI is used in velocity form, expression for which is derived below: By differentiating both sides we get,

$$\frac{du_{PI}(t)}{dt} = K_p \frac{de(t)}{dt} + K_I e(t) \quad (3.2)$$

or

$$\frac{[u_{PI}(k) - u_{PI}(k-1)]}{T_s} = K_p \frac{[e(k) - e(k-1)]}{T_s} + K_I e(k) \quad (3.3)$$

or

$$\Delta u_{PI}(k) = K_p \Delta e(k) + K_I e(k) \quad (3.4)$$

$u_{PI}(k)$  can be computed as:

$$\Delta u_{PI}(k) = \frac{[u_{PI}(k) - u_{PI}(k-1)]}{T_s} \quad (3.5)$$

$$u_{PI}(k) - u_{PI}(k-1) = T_s \Delta u_{PI}(k) \quad (3.6)$$

$$u_{PI}(k) = T_s \Delta u_{PI}(k) + u_{PI}(k-1) \quad (3.7)$$

Where,  $T_s$  is the sampling time.

Figure 3.2 shows the simulation diagram in which a PI Controller is designed for the depth control of the submarine. The PI controller used here is designed in MATLAB Simulink<sup>®</sup> with gains  $K_p = 0.6$  and  $K_I = 0.1$  which are assumed by Klee and Allen in [5] and the simulation results for the different Submarine parameters are shown in Figure 4.1.2 (Stern Plane Angle), Figure 4.1.3 (Rate of Change of Depth) and Figure 4.1.4 (Actual Depth). All these responses are observed and results are recorded in accordance with the convergence to  $0^\circ$  (Stern Plane Angle), 0 (Rate of Change of Depth) and to the set point (Actual Depth) in Table 4.4.

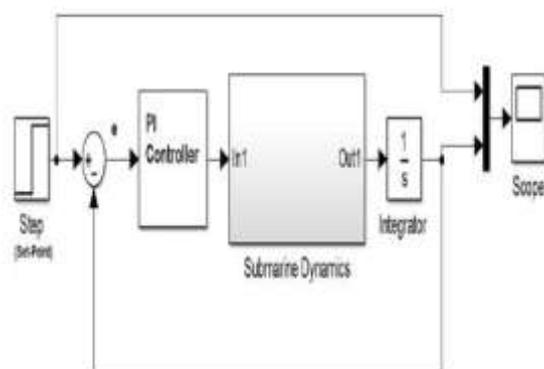


Figure 3.2 Depth Control of Submarine using PI Controller

### 3.2 Fuzzy Logic Controller

Fuzzy control is a practical alternative for a variety of challenging control applications since it provides a convenient method for constructing nonlinear controllers via the use of heuristic information. Such heuristic information may come from an operator who has acted as a “human-in-the-loop” controller for a process.

In the fuzzy control design methodology, we ask the operator to write down a set of rules on how to control the process, and then we incorporate these into a fuzzy controller that emulates the decision-making process of the human. In other cases, the heuristic information may come from a control engineer who has performed extensive mathematical modeling, analysis, and development of control algorithms for a particular process. Again, such expertise is loaded into the fuzzy controller to automate the reasoning processes and actions of the expert. Regardless of where the heuristic control knowledge comes from, fuzzy control provides a user-friendly formalism for representing and implementing the ideas we have about how to achieve high-performance control. Basic functional scheme of a FLC is given in Figure 3.2.

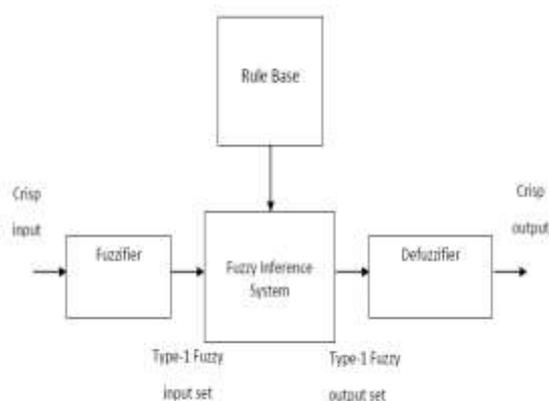


Figure 3.3 Basic Functional Scheme of FLC

A FLC is designed in velocity form to control the system. Figure 3.6 shows block diagram of the complete control strategy using FLC. The FLC structure consists of four main building blocks:

- The fuzzifier that maps crisp input either in the direct form or normalized form, i.e. input ranges between  $[-1, 1]$ , to the corresponding type-1 Fuzzy set,
- The “rule base” consists of a set of rules that depicts the knowledge of the designer about actions to be taken by the controller,
- The Fuzzy inference system (FIS), interpret the type-1 Fuzzy input set to type-1 Fuzzy output set according to rules provided in the rule base, and lastly,
- The defuzzifier convert type-1 Fuzzy output set of FIS to a crisp output. A type-1 Fuzzy logic controller is designed in velocity form to control Submarine Dynamics.

### 3.2.1 Fuzzification

Fuzzy sets are used to represent the information in the rule base, and FIS operates on these input Fuzzy sets and produces output Fuzzy sets; so a mechanism is required to convert a crisp input to corresponding Fuzzy set, this mechanism is known as “fuzzification”.

Let  $u_i$  is the crisp input such that  $u_i \in U_i$ , the universe of discourse, and  $U_i^*$  denotes the all possible Fuzzy sets that can be defined on  $U_i$ . Then the fuzzification operation, F can be defined as follows:

$$F: U_i \rightarrow U_i^*$$

The fuzzification transforms  $u_i$  to a Fuzzy set  $\tilde{A}_i$ , defined on the  $U_i$ , where

$$F_i = \tilde{A}_i \tag{3.8}$$

### 3.2.2 Linguistic variable

The main advantage of Fuzzy logic, as pointed by Zadeh [20] is that with linguistic variable and rule base, an operator can model his thinking. A linguistic variable can be characterized by following means:

- a) name of the variable,
- b) its linguistic Fuzzy sets,
- c) universe of discourse,
- d) “syntactic rule” of Fuzzy sets, and
- e) “Semantic rule” of Fuzzy sets.

Error (e) and rate of change of error ( $\dot{e}$ ) are two input linguistic variable, while controller output (u) is the only output linguistic variable. Each linguistic variable has a universe of discourse [-1, 1]. Seven MFs as shown in Figure 3.4 are defined for each linguistic variable i.e. Negative Large (NL), Negative Medium (NM), Negative Small (NS), ZERO (ZE), Positive Small (PS), Positive Medium (PM) and Positive Large (PL). Each membership function is a triangular function except for NB and PB which are trapezoidal functions.

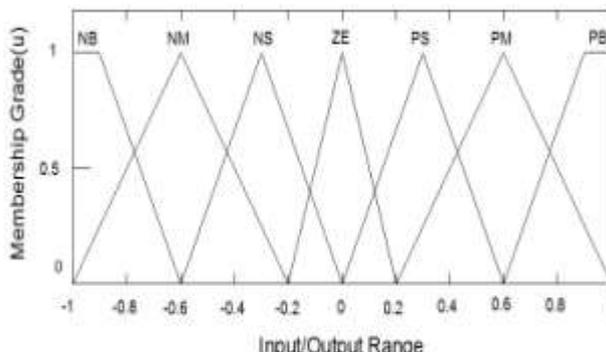


Figure 3.4 Type-1 Fuzzy sets for error (e), rate of change of error (ė) and control variable

Fuzzy if-then rule: In the simplest form, “Fuzzy if-then rule” can be represented as:

$$\text{If } x_1 \text{ is A and } x_2 \text{ is B then } y \text{ is C,}$$

Where A, B and C are linguistic values defined by Fuzzy sets on the universe of discourse  $X_1$ ,  $X_2$  and  $Y$ , respectively. The part between ‘if and then’ is called “antecedent” and the part after ‘then’ is called “consequent”.

### 3.2.3 Fuzzy Inference System

There are two types of FIS that are widely used in applications: Mamdani FIS [23] and Takagi-Sugeno-Kang (TSK) FIS [24]. The differences between these two FIS lie in the consequent part of their Fuzzy rule, and thus they have different defuzzification accordingly.

#### a) Mamdani FIS

Mamdani FIS was proposed as the first attempt to control a steam engine and boiler combination by a set of linguistic control rules obtained from experienced human operators. In Mamdani FIS the consequent part is also represented by Fuzzy set. Min-Max composition of Mamdani FIS is generally used. Figure 3.5 Shows how a two rule Mamdani FIS compute the overall output Fuzzy set when subject two crisp input  $x_1$  and  $x_2$ , using Min-Max composition of Mamdani FIS [23].

**b) TSK FIS**

The TSK FIS was proposed by Takagi, Sugeno and Kang with a view to develop a systematic approach for generating Fuzzy rules from input-output data set [24]. In TSK FIS, the consequent part is represented by a conventional a polynomial function. A typical TSK Fuzzy rule has the form:

$$\text{IF } x \text{ is } A \text{ and } y \text{ is } B \text{ then } z = f(x,y);$$

Where A and B are input Fuzzy sets in the antecedent and  $f(x,y)$  is a polynomial of input variable x and y, in the consequent. However  $f(x,y)$  can be any function, until the output defined by it is within the Fuzzy region specified by the antecedent of the rule.

In this paper, Mamdani FIS with Min-Max composition is used for inference mechanism. In this inference mechanism first we consider the minimum membership degree of various antecedents for each rule and, then the membership degree of output MF is calculated by taking a maximum membership degree of all the consequent consisting same MF. In a simple way it can be given as:

$$\mu_c(z) = \max[\min[\mu_A(\text{input}(i)), \mu_B(\text{input}(j))]] \tag{3.9}$$

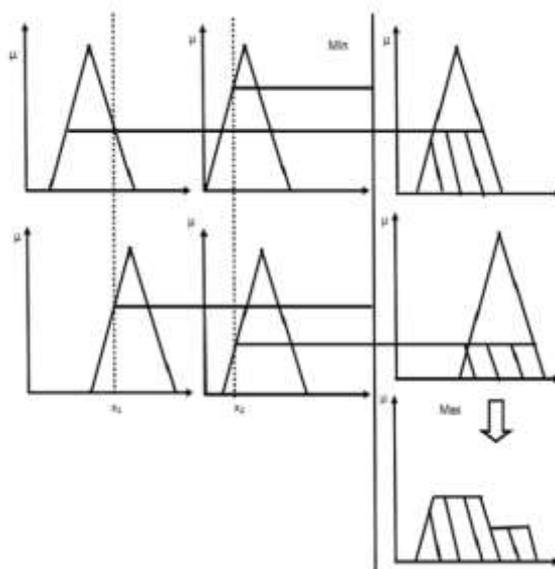


Figure 3.5 Mamdani FIS using Min and Max operation

**3.2.4 Rule Base**

The maximized rule base consisting of 49 rules is shown in Table 3.1.

**Table 3.1: Rule base table for FLC**

e\de	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NM	NS	NS	ZE
NM	NB	NM	NM	NM	NS	ZE	PS
NS	NB	NM	NS	NS	ZE	PS	PB
ZE	NB	NM	NS	ZE	PS	PM	PB
PS	NM	NS	ZE	PS	PS	PM	PB
PM	NS	ZE	PS	PM	PM	PM	PB
PB	ZE	PS	PS	PM	PB	PB	PB

**3.2.5 Defuzzification**

Defuzzification is a process to extract crisp output from the type-1 Fuzzy output set, provided by FIS. There are various defuzzification methods available in Control circles. Centroid-of-area defuzzification method has been used here and it is explained below:

Using centroid of area method the crisp output,  $Z_{COA}$  is given as:

$$Z_{COA} = \frac{\int_Z \mu_A(z)zdz}{\int_Z \mu_A(z)dz} \tag{3.10}$$

Where A is the output Fuzzy set to be defuzzified defined in the universe of discourse z,  $\mu_A(z)$  is the aggregated output of MFs.

The input and output variables of Fuzzy controller are normalized and de-normalized using various gains or scaling factors.

Figure 3.6 shows the simulation diagram in which an FLC is designed for the depth control of the submarine. The controller is used in velocity form working on error,  $e$  and rate of change of error  $\dot{e}$  and the simulation responses for the different Submarine parameters are shown in Figure 4.1 (Stern Plane Angle), Figure 4.2 (Rate of Change of Depth) and Figure 4.3 (Actual Depth). All these responses are observed and results are recorded in accordance with the convergence to  $0^\circ$  (Stern Plane Angle), 0 (Rate of Change of Depth) and to the set point (Actual Depth) in Table 4.3.

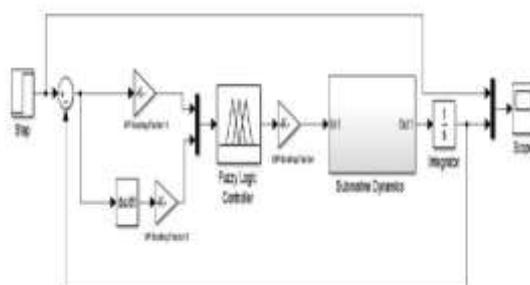


Figure 3.6 Depth Control of Submarine using the FLC

### 3.3 Type-2 Fuzzy Controller

T2FLC is a higher order of Fuzzy logic, in which type-2 Fuzzy sets are used, which have uncertainty about the MFs. As we know an FLS comprises of certain rules and in general, the knowledge used to build these rules is uncertain. In type-1 FLC, MFs are type-1 Fuzzy sets. Such sets are unable to handle uncertainty in linguistic knowledge. On the other hand, type-2 FLC are capable of handling such uncertainties in linguistic knowledge as they use type-2 Fuzzy sets. The concept of type-2 Fuzzy set was introduced by Zadeh [21]. A type-2 Fuzzy set is defined by a Fuzzy membership function, whose membership degree of each element of this Fuzzy set is another Fuzzy set in [0, 1], contrary to type-1 Fuzzy set where the membership degree is a crisp value in [0, 1]. The basic block diagram of type-2 FLC is shown in Figure 3.7. There are two kinds of FLC: (1) interval type-2 FLC and (2) Gaussian type-2 FLC. In the present work, Interval T2 FLC has been considered.

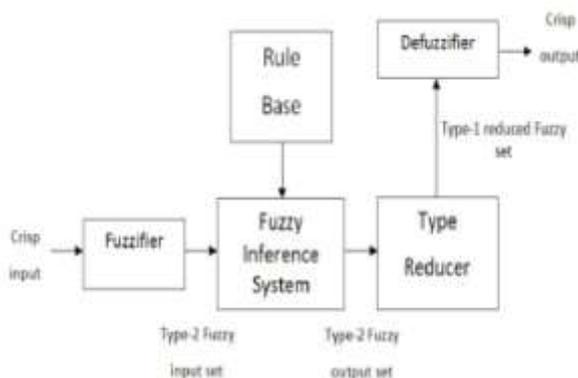


Figure 3.7 Block Diagram of T2FLC

The T2FLC used here is designed in velocity form and Figure 3.11 shows block diagram for the same performing the depth control. A T2FLC essentially consists of following functional elements:

#### 3.3.1 Fuzzification

The fuzzifier maps crisp Input into a type-2 Fuzzy set and in the present work the type-2 Fuzzy set used here is an interval type-2 Fuzzy set. It is also to be noted that in a T2FLC at least one membership function of either the antecedents or consequents belongs to type-2 Fuzzy sets.

**3.3.2 Linguistic Variables**

In T2FLC also, error (e) and rate of change of error (ė) are two input linguistic variables, while controller output (u) is the only output linguistic variable. Each linguistic variable has a universe of discourse [-1, 1]. UOD also consists of seven MFs for each input and output linguistic variable, as shown in Figure 3.8 and Figure 3.9 respectively.

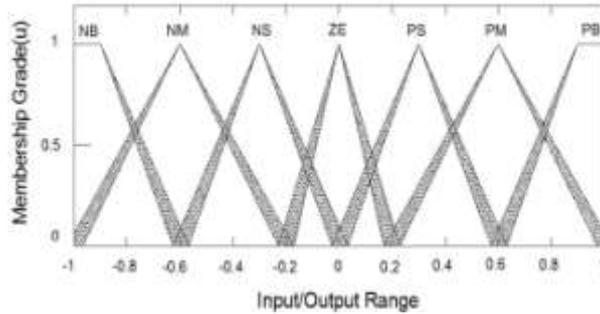


Figure 3.8 Type-2 Fuzzy sets for error (e) and rate of change of error (ė)

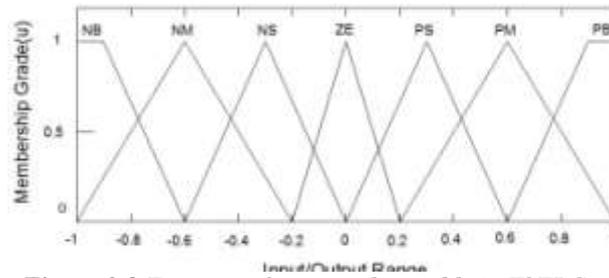


Figure 3.9 Fuzzy sets for control variable in T2FLC

The Fuzzy set used here is interval type-2, and each type-2 Fuzzy set comprises of two Triangular shape type-1 Fuzzy sets bounding footprint of uncertainty.

**3.3.3 Fuzzy Inference System**

In type-2 FLS, the FIS gives a mapping from input type-2 Fuzzy sets to output type-2 Fuzzy sets using rule base. In an interval type-2 FLS, intersection under minimum operation is performed and it results in an interval type-1 Fuzzy set,  $F^l$  which can be represented as:

$$F^l = [\underline{f}^l, \overline{f}^l] \tag{3.11}$$

Where  $\underline{f}^l, \overline{f}^l$ , for two inputs can be defined as:

$$\underline{f}^l(z) = \min \left[ \underline{u}_{F_i^l}(input(i)), \underline{u}_{F_j^l}(input(j)) \right] \tag{3.12}$$

$$\overline{f}^l(z) = \min \left[ \overline{u}_{F_i^l}(input(i)), \overline{u}_{F_j^l}(input(j)) \right] \tag{3.13}$$

Pictorial representation of intersection under minimum operation for type-2 FLS [22] is shown in Figure 3.10.

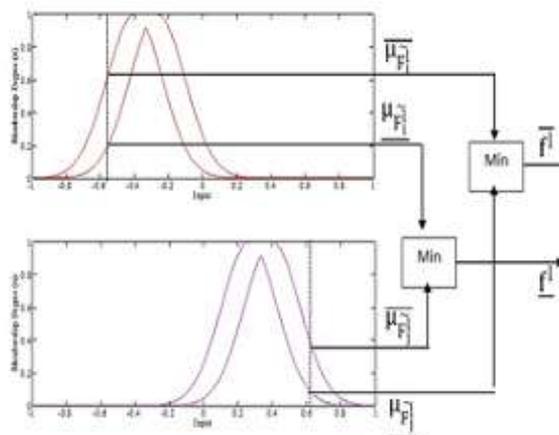


Figure 3.10 T2FLS intersection under minimum operation

### 3.3.4 Rule Base

The structure of rules in a type-1 FLS and type-2 FLS is the same, the only difference is that in the type-2 FLS the antecedents and the consequents are membership function of type-2 Fuzzy sets. The typical structure of a type-2 FLS rule can be represented as:

If  $x_1$  is  $\tilde{A}$  and  $x_2$  is  $\tilde{B}$  then  $y$  is  $\tilde{C}$ ,

Where  $\tilde{A}$ ,  $\tilde{B}$  and  $\tilde{C}$  are appropriate type-2 Fuzzy sets.

Type-2 FLS also uses same set of maximized rule base consisting of 49 rules are shown in Table 3.2.

**Table 3.2: Rule base table for T2FLC**

e\de	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NM	NS	NS	ZE
NM	NB	NM	NM	NM	NS	ZE	PS
NS	NB	NM	NS	NS	ZE	PS	PB
ZE	NB	NM	NS	ZE	PS	PM	PB
PS	NM	NS	ZE	PS	PS	PM	PB
PM	NS	ZE	PS	PM	PM	PM	PB
PB	ZE	PS	PS	PM	PB	PB	PB

### 3.3.5 Type Reducer

As the output obtained from type-2 FIS are type-2 Fuzzy sets, it is needed to convert it to a type-1 Fuzzy set which can be further converted to a crisp output using defuzzification. So, a type reducer operation reduces type-2 Fuzzy sets to a type-1 Fuzzy set known as “type-reducer set”.

There are various methods of type-reduction, such as centroid, center-of-sum, center-of-sets, height, and modified height. In the present work, center-of-sum has been used, in a simple form which can be mathematically expressed as:

$$y_{cos} = [y_l \ y_r] \tag{3.14}$$

Where,  $y_l$  and  $y_r$  are respectively the maximum and minimum value of  $y_{cos}$  each of them can be expressed using a Fuzzy basis function as:

$$y_l = \frac{\sum_{i=1}^M f_l^i y_l^i}{\sum_{i=1}^M f_l^i} \tag{3.15}$$

$$y_r = \frac{\sum_{i=1}^M f_r^i y_r^i}{\sum_{i=1}^M f_r^i} \tag{3.16}$$

Where,  $y_l^i, y_r^i$  are centroid of the type-2 interval consequent set  $\tilde{G}^i$  (the centroid of type-2 Fuzzy set are derived according to methods given in [25, 26].

$f_l^i, f_r^i$  Represent the firing strength membership degree contributing to the left-most point  $y_l^i$  and the right-most point  $y_r^i$ .

Centre-of-sets type-reduction method is used in the present work. It was proposed by Karnik and Mendel [25, 26].

### 3.3.6 Defuzzification

As  $y_{cos}$  is an interval set, defuzzification is done to obtain a crisp output of an interval type-2 FLS. Defuzzification in done using average of  $y_l^i$  and  $y_r^i$ , represented as:

$$y(x) = \frac{y_l + y_r}{2} \tag{3.17}$$

A type-2 Fuzzy controller is also designed in velocity form to control Submarine Dynamics and Figure 3.11 shows a block diagram for the same using type-2 FLC. Crisp output of type-2 Fuzzy is obtained by averaging method, given by Eqn. 3.18, variables carries their meaning as defined in previous chapters.

$$y(x) = \frac{y_l + y_r}{2} \tag{3.18}$$

The input and output variable of type-2 Fuzzy controller are normalized and denormalized using various gains or scaling factors.

Figure 3.11 shows the simulation diagram in which a T2FLC is designed for the depth control of the submarine. The controller is used in velocity form working on error,  $e$  and rate of change of error  $\dot{e}$  and the simulation responses for the different parameters are shown in Figure 4.1 (Stern Plane Angle), Figure 4.2 (Rate of Change of Depth) and Figure 4.3 (Actual Depth). All these responses are observed and results are recorded in accordance with the convergence to  $0^\circ$  (Stern Plane Angle), 0 (Rate of Change of Depth) and to the set point (Actual Depth) in Table 4.3.

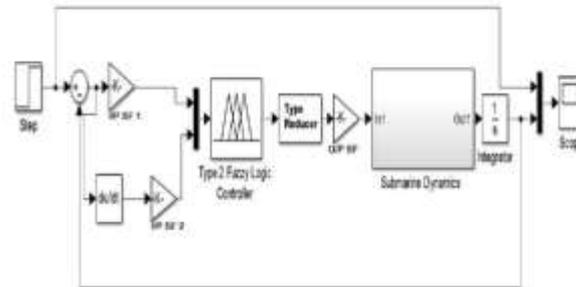


Figure 3.11 Depth Control of Submarine using T2FLC

#### IV. Performance Evaluation & Discussion of Results

##### 4.1 Performance Evaluation

The next step in the design process is to perform analysis and performance evaluation. Basically, we need performance evaluation to test that the control system that we design does in fact meet the closed-loop specifications (e.g., for “commissioning” the control system). This can be particularly important in safety-critical applications such as a nuclear power plant control, in aircraft control [18] or the one considered here; Depth Control of Submarine. To compare the results of various controllers, a number of performance measures can be used. These performance measures give a quantitative valuation of various controllers. Some of these performance measures are given below:

##### 4.1.1 Settling time, $t_s$

Settling time is defined as the time required for the response curve to reach and stay within a range of certain percentage (usually 5% or 2%) of the final value. Settling time depends on the system response and time constant. Mathematically stating,

$$t_s = 0.02(\text{or } 0.05 \text{ for } 5\%) \times \text{reference value}$$

##### 4.1.2 Rise time, $t_r$

In electronics, when describing a voltage or current step function, rise time is the time taken by a signal to change from a specified low value to a specified high value. For applications in control theory, rise time is defined as “the time required for the response to rise from  $x\%$  to  $y\%$  of its final value”, with 0% to 100% rise time common for underdamped second order systems, 5% to 95% for critically damped and 10% to 90% for overdamped ones. For a given system output, its rise time depend both on the rise time of input signal and on the characteristics of the system.

##### 4.1.3 Overshoot, $M_p$

Overshoot is defined as the difference between the first peak of the response and the reference or desired output of the system. Mathematically, this can be stated as

$$M_p = \frac{\text{first peak of response} - \text{desired value}}{\text{desired value}} \times 100$$

##### 4.1.4 Steady state error, $e_{ss}$

Steady state error is defined as the constant error that persists in the steady state of operation, i.e., the state when the response becomes constant and do not change with time. It may be defined as the difference between the desired value of the response and the actual value at steady state.

$$e_{ss} = \text{desired value} - \text{actual value},$$

at steady state. Figure 5.1 shows diagrammatically some of the performance measures for step response of a second order system.

#### 4.1.5 Integral Square error (ISE)

Integral square error is one of the performance measures to quantify the performance of a controller. As its name suggests, Integral square error is the integral of the error squared, i.e.,

$$ISE = \int e^2 dt ;$$

Where  $e$  is the error defined as,  $e = \text{desired value} - \text{actual value}$

#### 4.1.6 Integral Absolute Error (IAE)

Integral Absolute error is, again, one of the performance measures that give quantification of the controller's performance. Mathematically, it is given as

$$IAE = \int |e(t)| dt ;$$

i.e., IAE is the integral of the absolute error.

#### 4.1.7 Integral Time Absolute Error (ITAE)

Integral time absolute error is defined as the integral of time factor multiplied by absolute error between the actual value of the response and the desired value, i.e., ITAE may be given as,

$$ITAE = \int t|e(t)|dt$$

The selection of the criteria depends on the type of response desired, the errors will contribute different for each criterion, so we have that large errors will increase the value of ISE more heavily than to IAE. ISE will favor responses with smaller overshoot for load changes, but ISE will give longer settling time. In ITAE, time appears as a factor, and therefore, ITAE will penalize heavily, those errors that occur late in time, but virtually ignores errors that occur early in time. Designing using ITAE will give us the shortest settling time, but it will produce the largest overshoot among the three criteria considered. Designing considering IAE will give us an intermediate result; in this case, the settling time will not be so large than using ISE or so small than using ITAE and the same applies for the overshoot response. The selection of a particular criterion is depending on the type of desired response [19]. The Quantitative analysis in terms of Performance Criteria of the various controllers used in this report has been carried out in Table 4.2.

## 4.2 Discussion of Results

In the present work, Submarine dynamics for Depth control movement has been simulated in MATLAB Simulink environment. For this purpose, some widely used Controllers viz. PI Controller, FLC and T2FLC have been implemented. A comparison between all the controllers used was accomplished and is shown in Table 4.2. Overall results of simulation in terms of time response have been observed by virtue of three parameters viz. Stern plane angle (convergence to  $0^\circ$ ), Rate of change of Depth (convergence to 0) and Actual Depth (convergence to Commanded depth or SP) and are shown in figures 4.1, 4.2 and 4.3 respectively. Figure 4.1 shows the plot showing Stern Plane Angles achieved by various controllers implemented in this paper; the angle of the submarine at first rises abruptly and reaches a peak value and then slowly converges to  $0^\circ$  in steps indicating that the stern plane of the submarine is getting settled horizontally and this occurs as the submarine achieves the commanded depth or the set point. Stern Plane Angle is the output of the controller in the loop for the Depth Control Process. The time at which the controller achieves the convergence to  $0^\circ$ , i.e.,  $t_0$ , is recorded. The peak angle achieved by the Stern plane and the time at which it converges with  $0^\circ$  for the various controllers used in this paper are given in tabulated form in Table 4.1.

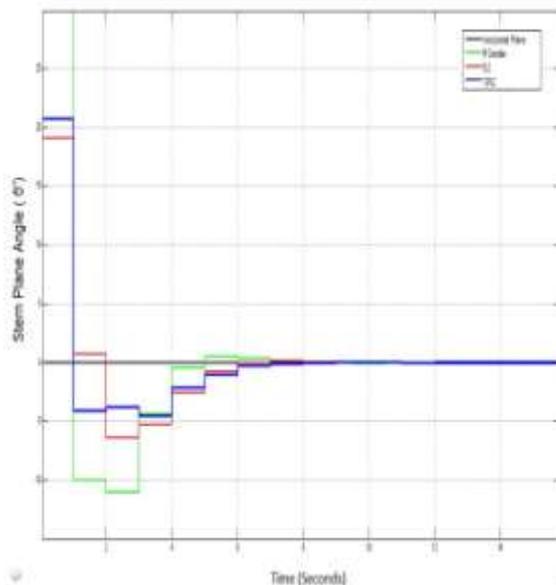


Figure 4.1 Stern Plane angle achieved by various controllers

Table 4.1 Peak Stern Plane Angle achieved by various controllers

Type of Controller	Peak Stern Angle (θ°)
PI Controller	30°
FLC	19°
T2FLC	21°

Figure 4.2 shows the Rate of change of the Depth which rises initially but after some time decreases and converges to 0 as the submarine achieves the commanded depth. The time taken by the controller for convergence of Rate of Change of Depth,  $t_v$  is recorded and the time at which it converges with 0 for the various controllers used in this paper are given in tabulated form in Table 4.3.

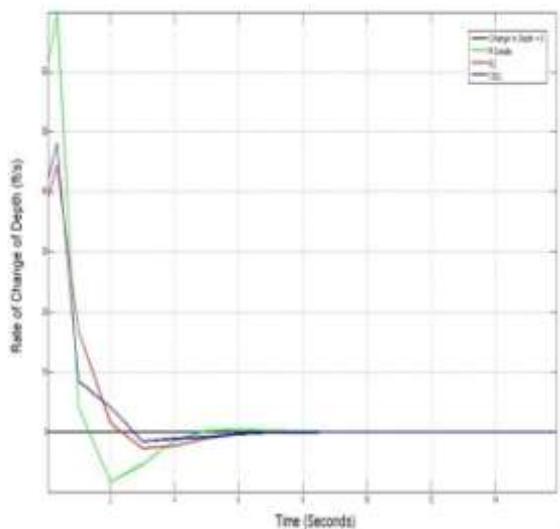


Figure 4.2 Rate of Change of Depth achieved by various controllers

Figure 4.3 shows the actual depth response of the controller. The Set-point (Commanded depth) is taken as 50 feet (Periscopic Depth) and it is observed here that the response rises abruptly in the start which is due to the baseline parameters of the Submarine Dynamics namely  $K_{\theta}$  and  $K_{\dot{\theta}}$  and then slowly it converges with the Commanded depth and the time it converges with the set point,  $t_s$  is recorded along with other parameters such as Rise time,  $t_r$ ; Peak,  $p$ ; Overshoot,  $M_p$  and Steady state error,  $e_{ss}$  in Table 4.2.

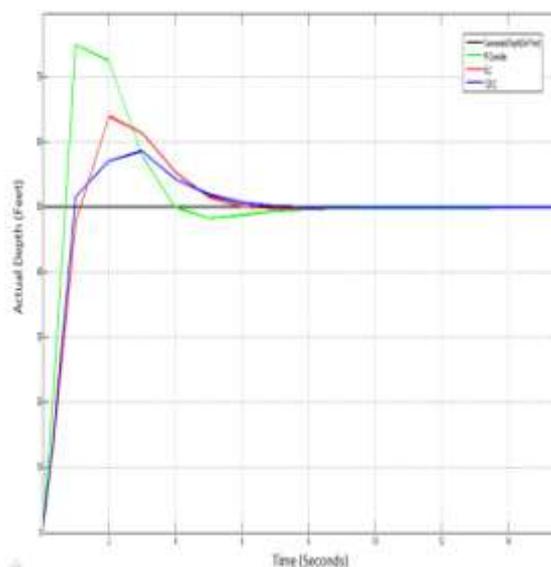


Figure 4.3 Actual Depth achieved by Various controllers

Table 4.2 Quantitative Analysis of Controllers in terms of Performance Criteria

Performance Criteria	Type of Controller		
	PI Controller	FLC	T2FLC
Settling Time, $t_s$ (s)	6.3	5.5	5.6
Rise Time, $t_r$ (s)	0.6	1.1	0.9
Peak, $p$	75	63.6	58.9
Overshoot, $M_p$	50%	27.2%	17.8%
Steady State Error, $e_{ss}$	0.02	-	-
ISE	3696	2905	2680
IAE	110	90	76
ITAE	189	158	115

The results show that the T2FLC has better performance than the other controllers when analyzed in terms of various performance criteria and Submarine Parameters and following observations has been made from Tables 4.2 and 4.3:

In terms of Performance Criteria, it is observed that the Settling time,  $t_s$  is largest for PI controller, smallest for FLC. Rise time is greatest in FLC whereas it is smallest in PI controller; T2FLC's rise time is greater than PI controller. Peak and Peak overshoot is greatest in PI controller, smallest in T2FLC and FLC is giving performance in between PI Controller and T2FLC's. There is a very small Steady state error in PI controller whereas it is nil for FLC and T2FLC. In terms of ISE, PI controller is having greatest ISE; T2FLC is giving the smallest ISE and it is in between PI controller and T2FLC for FLC. IAE is greatest for PI Controller, smallest for T2FLC and it is in between PI controller and T2FLC for FLC. ITAE is largest in PI controller, smallest in T2FLC and it is in between PI controller and T2FLC for FLC.

**Table 4.3 Comparison of Results in terms of Submarine Parameters**

Type of Controller	Submarine Parameter		
	Convergence of Stern Plane Angle ( $\theta$ ), $t_\theta$ (s)	Convergence of Rate of Change of Depth ( $v$ ), $t_v$ (s)	Actual Depth (c), $t_s$ (s)
PI Controller	12	14.6	6.32
FLC	13	12.8	5.54
T2FLC	9	9	5.69

In terms of Submarine parameters, Stern plane Angle in all the controllers rises abruptly and then slowly converges to  $0^\circ$  (longitudinal axis of submarine) in steps indicating that the stern plane of the submarine is getting settled horizontally and this occurs as the submarine achieves the commanded depth. This convergence time,  $t_\theta$  is largest for PI controller and smallest for T2FLC. Rate of change of the Depth is also decreases and converges to 0 as the submarine achieves the commanded depth and this convergence time,  $t_v$  is largest in PI controller and smallest in T2FLC. In the Actual depth response of all the controllers it is to be noted that the response shoots in the start which is contribution of the constant parameters of the Submarine Dynamics namely  $K_\theta$  and  $K_\delta$  and also the convergence to set point or commanded depth in this case is measured in form of Settling time,  $t_s$  which is largest for PI Controller and smallest for FLC, also, the difference between settling time of other controllers is very small hence we can conclude that T2FLC is giving the best performance among the controllers. FLC's performance is better than PI controller.

## V. Conclusion and Future Scope

### 5.1 Conclusion

In this paper, depth control of a submarine has been studied. A comparative study based on various controllers viz. PI Controller, Type 1 Fuzzy logic based controller and Type 2 Fuzzy logic based controllers has been done in MATLAB Simulink environment. The effect of environmental disturbances is left out in this paper and all simulations done are without any consideration of these disturbances on the Submarine. MATLAB Simulink is a flexible and powerful program. With the recorded results two comparisons have been accomplished. In the first comparison shown in Table 4.2 performance of these controllers has been analyzed in terms of common performance criteria and in Table 4.3 performances of these controllers has been observed in terms of Submarine Parameters and an implication can be drawn from both these comparisons that T2FLC is giving better control than the other controllers.

Following conclusions can be drawn from the Results:

- a) T2FLC has better performance than the other controllers when analyzed in terms of various performance criteria and Submarine Parameters observed with time response specifications.
- b) Stern plane Angle in all the controllers rises abruptly and then slowly converges to  $0^\circ$  in steps indicating that the stern plane of the submarine is getting settled horizontally and this occurs as the submarine achieves the commanded depth.
- c) Rate of change of the Depth is also decreases and converges to 0 as the submarine achieves the commanded depth.
- d) In the actual depth response of all the controllers it is to be noted that the response shoots in the start which is due to the inherent parameters of the Submarine Dynamics namely  $K_\theta$  and  $K_\delta$ .

### 5.2 Future Scope

Controllers used in this paper can be optimized using various Optimization techniques such as Ant-colony optimization, Genetic Algorithm, Particle Swarm Optimization, Biogeography Based optimization etc. In this paper all the observation are made without taking into account the effect of Hydrodynamic and Hydrostatic Forces occurring in the sea environment acting on a body submerged and/or operating in the sea, such as Hydrodynamic Force, Radiation Force, Excitation Force And Drag Force. Also, in this paper all the observation are made without taking into account the effect of various disturbances occurring in the sea environment acting on a body submerged and/or operating in the sea. Some methods to generate the disturbances caused by the waves are listed below:

- a) The Bretschneider Spectrum
- b) The Pierson and Moskowitz Spectrum

c) The JONSWAP Spectrum

**References**

- [1] D.K. Brown, "The technology of Submarine design – a historical survey 1905-1945", *Interdisciplinary Science Reviews*, Taylor and Francis, 1990, 15(3), 235-251.
- [2] R.F. Burges, "Ships beneath the Sea: A History of Submarines and Submersibles", McGraw-Hill, New York, 1975.
- [3] A.G. Bovis, "Optimal control, safety and silence- Trends in design of maneuvering systems for French submarines", *Proceedings of Warship 88 International Symposium on Conventional Naval Submarines*, London, UK, 1988.
- [4] W. Braugnier, "Automatic depth control of submarine split plane mode", *Naval Ship Systems Command Technical News* 15, part 7, July, 1966.
- [5] H. Klee and R. Allen, "Simulation of Dynamic Systems with MATLAB and Simulink", 2<sup>nd</sup> Edition, Boca Raton, Florida, USA, CRC press, Taylor and Francis Group, 2011, 85-89.
- [6] M. J. Grimble, G.M. Van Der Molen and E. Liceaga-Castro, "Submarine Depth and Pitch Control", 2<sup>nd</sup> IEEE Conference on Control Applications, September 1993, Vancouver, Canada, 953-958.
- [7] M. Ekici, H. Kahveci and A.S. Akpınar, "A LabVIEW based submarine depth control simulator with PID and Fuzzy controller", *Innovations in Intelligent systems and applications (INISTA)*, 2013, Albena, 1– 6.
- [8] R. Xu and C. Kwan, "Submarine Pitch and Depth control using FCMAC neural Networks", *Proceedings of the American Control Conference*, July 1998, Philadelphia, Pennsylvania, USA, 379-383.
- [9] E. Liceaga-Castro and G.M. Van der Molen, "Submarine H<sup>∞</sup> depth control under wave disturbances", *IEEE Transactions on Control Systems Technology*, Vol. 3 , No. 3, 1995, 338 – 346.
- [10] E. Liceaga-Castro and G.M. Van Der Molen, "A submarine Depth Control system design", *International Journal of Control*, Taylor and Francis, 1995, 61:2, 279-308.
- [11] Y. Hao, D. Shen and Z. Xiong, "Design of Submarine near Surface Depth Controller", *Proceedings of the 5<sup>th</sup> world congress on intelligent control and automation*, June 2004, Hangzhou, China, 4530-4533.
- [12] L. Lapiere, "Robust Diving control of an AUV", *Ocean Engineering* 36, Elsevier Ltd., 2009, 92-104.
- [13] G.M. Van der Molen and M.J. Grimble, "H<sup>∞</sup> submarine depth and pitch control", *IEEE Conference on Decision and Control*, 1993, Dec 1993, vol .3, 2511 – 2516.
- [14] S.K. Lee, K.H. Sohn, S.W. Byun and J.Y. Kim, "Modeling and controller design of Manta-type unmanned underwater test vehicle", *Journal of Mechanical science and technology* 23, Springer-verlag, 2009, 987-990.
- [15] E. Liceaga-Castro, J. Liceaga-Castro, C.E. Ugalde-Loo and E.M. Navarro-Lopez, "Efficient multivariable Submarine depth control system design", *Ocean Engineering*, 35, 2008, 1747-1758.
- [16] K.H. Low, "Locomotion and Depth Control of Robotic fish with modular undulating Fins", *International Journal of Automation and Computing* 4(2006), 348-357.
- [17] R. Allen, R.K. Lea and S.L. Merry, "A comparative study of control techniques for an underwater flight vehicle", *International Journal of System Science*, Taylor and Francis, 1999, 30(9), 947-964.
- [18] K.M. Passino and S. Yurkovich, "Fuzzy Control", Addison-Wesley Longman, Inc., 1998.
- [19] O. Castillo and P. Melin, "Type-2 Fuzzy logic: theory and applications", *Studies in Fuzziness and Soft Computing*, vol. 223; Heidelberg: Springer; 2008.
- [20] L.A. Zadeh, "Fuzzy sets", *Information and Control* 1965; 8, 338-353.
- [21] L.A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning—I", *Information Sciences* 1975; 8, 199–249.
- [22] O. Castillo and P. Melin, "Soft computing for control of non-linear dynamical systems", 1<sup>st</sup> ed. Heidelberg: Springer; 2001.
- [23] E.H. Mamdani and S. Assilian, "An experimental in linguistic synthesis with a Fuzzy logic controller", *International Journal of Man-Machine Studies* 1975; 7(1), 1-13.
- [24] T. Takagi and M. Sugeno, "Fuzzy Identification of Systems and its application to modeling and control", *IEEE Transactions on Systems Man and Cybernetics* 1985; 15(1), 116-132.
- [25] N.N. Karnik and J.M. Mendel, "An introduction to type-2 Fuzzy logic systems", USC Report, <http://sipi.usc.edu/~mendel/report>, Oct. 1998.
- [26] N.N. Karnik and J.M. Mendel, "Type-2 Fuzzy logic systems: type-reduction", *IEEE International Conference on Systems Man and Cybernetics*, San Diego, CA, Oct. 1998.

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